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Seat No. _______________

HK-003-1014008

B. Sc. (Sem. IV) (CBCS) (W.E.F. 2016) Examination

April - 2023

Mathematics : Paper - IV (A)

(Linear Algebra & Differential Geometry - 2016)

Faculty Code : 003
Subject Code : 1014008

Time : $2\frac{1}{2}$ Hours / Total Marks : **70**

Instructions : (1) All five questions are compulsory.
(2) Figures to the right indicate marks of corresponding question.

1 (a) Answer the following questions briefly : **4**

(1) Define Linear independence.

(2) Define subspace of a vector space.

(3) Show that the vectors (1, -2) and (-5, 10) are linearly dependent in R^2.

(4) Let $W = \{(1,-2,5),(2,1,-1),(3,-1,a)\}$ be Linearly Dependent subset of vector space R^3 then find a.

(b) Answer any one of the following : **2**

(1) For what value of m, the vector (m, 3, 1) is linear combination of (3, 2, 1) and (2, 1, 0)?

(2) Show that the set of vectors $\{(2,1,0,3),(3,-1,5,2),(-1,0,2,1)\}$ is Linearly Independent.

(c) Answer any one of the following : **3**

(1) Prove that W_1 and W_2 are sub space of vector space V then $W_1 \cap W_2$ is also a sub space of V.

(2) Show that the sub set $W_1 = \{(a, b, c) / 2a + 5b - c = 0\}$ is sub space of R^3.

(d) Answer any one of the following : **5**

(1) Check $V = \{(x, y) / x, y \in R\}$ is vector space or not under addition and scalar multiplication of $\alpha \in R$ as $(x_1, y_1),\ (x_2, y_2) \in V,\ (x_1, y_1) + (x_2, y_2) =$ $(x_1 + x_2, y_1 + y_2)$ and $\alpha(x_1, y_1) = (\alpha x_1, y_1)$, $x_1, x_2, y_1, y_2 \in R$.

(2) If vector $\overline{v}_k, (1 \le k \le n)$ of set $\{\overline{v}_1, \overline{v}_2, \overline{v}_3, ..., \overline{v}_n\}$ is linear combination of remaining vectors $\overline{v}_1, \overline{v}_2, ..., \overline{v}_{k-1}, \overline{v}_{k+1}, ... \overline{v}_n$ then prove

$$SP\{\overline{v}_1, \overline{v}_2, \overline{v}_3, ..., \overline{v}_n\} =$$
$$SP\{\overline{v}_1, \overline{v}_2, ..., \overline{v}_{k-1}, \overline{v}_{k+1}, ..., \overline{v}_n\}$$

2 (a) Answer the following questions briefly : **4**

(1) Define Base.

(2) Define Dimension of vector space.

(3) Write standard base of $P_3(R)$.

(4) The set $A = \{\overline{v}_1, \overline{v}_2, \overline{v}_3, ..., \overline{v}_n\}$ is base of vector space V then A is Linearly Independent. (True/ false).

(b) Answer any one of the following : **2**

(1) Show that the set {(1,0,0,0), (-1,1,1,0), (0,1,0,1)} of R^4 is Linearly Independent.

(2) If the set $W = \{(c+d, c+d, c, d) / c, d \in R\}$ of R^4 then find SpW, $\dim W$.

(c) Answer any one of the following : **3**

(1) Extend set A = {(1,1,1), (2,0,0)} of vector space R^3 to form a base of R^3.

(2) Show that the set $\{5, -6x + x^2, 9 - 6x + x^2\}$ is not a base of $P_2(R)$.

(d) Answer any one of the following : **5**

(1) Prove that set A = {(1, 2, 1), (2, 1, 0), (l, -1, 2)} forms a base of R^3. Find co-ordinates of (1, 1, -1) with respect to this base.

(2) Every Linearly Independent set of vector space V can be extended to form a basis.

3 (a) Answer the following questions briefly : **4**

(1) Define Linear Transformation.

(2) Define Kernel of Linear transformation.

(3) The linear transformation $T : U \to V$ is one-one if and only if $R(T) = \{\theta\}$. (True/False)

(4) Prove that linear transformation $T : R^3 \to R^3$, defined as $T(x, y, z) = (0, y, z)$ $\forall (x, y, z) \in R^3$ is idempotent.

(b) Answer any one of the following : **2**

(1) Let θ and θ' be zero vector of vector spaces U and V respectively. $T:U \to V$ is linear transformation then prove that $T(\theta) = \theta'$.

(2) Let $T:R^2 \to R^2, T(x, y) = (x+1, y-2), \forall (x, y) \in R^2$ then T is not linear transformation.

(c) Answer any one of the following : **3**

(1) Let $T:R^3 \to R^3$, $T(x, y, z) = (x-y, y-z, z-x)$, $\forall (x, y, z) \in R^3$ then T is linear transformation.

(2) Let $T:R^3 \to R^3$, is such that $T(a, b, c) = (a-b+c, b-c, c), \forall (a, b, c) \in R^3$, then find $N(T)$.

(d) Answer any one of the following : **5**

(1) Find the linear transformation $T:R^3 \to R^2$, such that $T(1, 0, 0) = (1, 1)$, $T(1, 1, 0) = (1, 0)$, $T(1, 1, 1) = (1, -1)$.

(2) Prove that a linear transformation $T:U \to V$ is one-one if and only if $N(T) = \{\theta'\}$.

4 (a) Answer the following questions briefly : **4**

(1) Define Eigen value of a linear transformation.

(2) If dim $U = m$ and dim $V = n$ then dim $\{L(U,V)\}$=____

(3) Define adjoint of a linear transformation.

(4) Find the Eigen value of matrix $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$.

(b) Answer any one of the following : **2**

(1) For linear transformation $T : R^2 \to R^2$;

$T(x, y) = (2x - y, x + 2y)$, Find $[T; B]$ where B is standard base.

(2) Find the characteristic equation of a matrix

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}.$$

(c) Answer any one of the following : **3**

(1) Let $T : V \to V$ be a linear transformation and B is any basis of V then λ is Eigen value if and only if

$$del\left(\left[\left(T - \lambda I_n\right), B\right]\right) = 0.$$

(2) For linear transformation $T : R^2 \to R^2$;

$T(x, y) = (x - 3y, 3x + 2y)$, find $[T; B]$ where B is standard base.

(d) Answer any one of the following : **5**

(1) If $T : R^2 \to R^2; T(x, y) = (x, -y)$ and

$B_1 = \{(1,1), (1,0)\}$ $B_2 = \{(2,3), (4,5)\}$, then find

$[T; B_1, B_2]$.

(2) If $T : R^2 \to R^2; T(x, y, z) = (2x + y - z, 3x - 2y + 4z)$;

$\forall (x, y, z) \in R^2$ and the basis are

$B_1 = \{(1,1,1), (1,1,0), (1,0,1)\}$, $B_2 = \{(1,3), (1,4)\}$

then find $[T; B_1, B_2]$

5 (a) Answer the following questions briefly : **4**

(1) Define point of inflexion.

(2) Define conjugate point.

(3) Define singular point of a curve.

(4) Find the radius of curvature of curve $s = c \log \sec \psi$.

(b) Answer any one of the following : **2**

(1) Find the radius of curvature of curve $y^2 = 12x$ at point (0, 0).

(2) Find the asymptotes parallel to co-ordinates of the curve

$$x^4 + x^2y^2 - a^2\left(x^2 + y^2\right) = 0$$

(c) Answer any one of the following : **3**

(1) Find the radius of curvature of curve

$y^4 + x^3 + a\left(x^2 + y^2\right) - a^2y = 0$ at origin.

(2) Find singular points of the curve $(x-1)^2 - y(y-1)^2 = 0$ and discuss its nature.

(d) Answer any one of the following : **5**

(1) Obtain the formula of radius of curvature for a curve $y = f(x)$.

(2) Find the radius of curvature of curve $r^2 = a^2 \cos 2\theta$.